Chromomagnetic stability of the CFL phase with a meson current

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We show that the color-flavor locked (CFL) phase of dense quark matter is unstable with respect to the formation of a Goldstone boson current in the vicinity of the point $m_s^2 = 2\mu\Delta$. In this Goldstone boson current phase all components of the magnetic screening mass are real.

1. INTRODUCTION

It is well known that the groundstate of three flavor quark matter at very high baryon density is the color-flavor-locked (CFL) phase [1]. At somewhat lower densities it is necessary to take into account a finite strange quark mass, which leads to gapless fermions in the spectrum if $\mu_s > \Delta$ (with $\mu_s = \frac{m_s}{2\mu}$) [2]. The problem is that these gapless fermion modes cause instabilities in current-current correlation functions [3, 4]. In the present contribution (which is based on [5]) we show that for $\mu_s \sim \Delta$ the instability is resolved by the formation of a non-zero Goldstone boson current. Similar currents were considered previously in [6, 7]. For large μ_s (when the gaps become small) it has been argued that the instability is resolved by the formation of a LOFF state [8].

In this work we ignore homogeneous kaon condensation [9], which simplifies the calulation of the dispersion laws.

2. FREE ENERGY

We consider an effective lagrangian that describes the interaction of gapped fermions with background gauge fields [7, 10]

$$\begin{aligned}\mathcal{L} \;=\; & \mathrm{Tr}\left(\chi_L^\dagger(iv\cdot\partial - \hat{\mu}^L - A_e Q)\chi_L\right) + \mathrm{Tr}\left(\chi_R^\dagger(iv\cdot\partial - \hat{\mu}^R - A_e Q)\chi_R\right)\\ & - i\mathrm{Tr}\left(\chi_L^\dagger\chi_L X v\cdot(\partial - iA^T)X^\dagger\right) - i\mathrm{Tr}\left(\chi_R^\dagger\chi_R Y v\cdot(\partial - iA^T)Y^\dagger\right)\\ & - \frac{1}{2}\sum_{a,b,i,j,k}\Delta_k\left(\chi_L^{ai}\chi_L^{bj}\epsilon_{kab}\epsilon_{kij} - \chi_R^{ai}\chi_R^{bj}\epsilon_{kab}\epsilon_{kij} + h.c.\right). \qquad (1)\end{aligned}$$

Here, $\chi_{L,R}^{ai}$ are left/right handed fermions with color index a and flavor index i, A_μ are $SU(3)_C$ color gauge fields, and $\hat{\mu}^L = MM^\dagger/(2\mu)$, $\hat{\mu}^R = M^\dagger M/(2\mu)$ are effective chemical potentials induced by the quark mass matrix M. The matrix $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ is the quark charge matrix and A_e is an electro-static potential. The fields X, Y determine the

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flavor orientation of the left and right handed gap terms and transform as $X \to LXC^T$, $Y \to RYC^T$ under $(L,R) \in SU(3)_L \times SU(3)_R$ and $C \in SU(3)_C$, and Δ_k $(k = 1,2,3)$ are the CFL gap parameters.

We will assume that $X = Y = 1$ which excludes the possibility of kaon condensation. This assumption significantly simplifies the calculation of the fermion dispersion relations and the current correlation functions. It is possible to suppress kaon condensation by including a large instanton induced interaction [11], which does not change our results qualitatively [5].

We wish to study the possibility of forming a Goldstone boson current. Gauge invariance implies that the free energy only depends on the combinations $\vec{J}_L = X(\vec{\nabla} - i\vec{A}^T)X^\dagger$ and $\vec{J}_R = Y(\vec{\nabla} - i\vec{A}^T)Y^\dagger$. We will restrict ourselves to diagonal currents $\vec{J}_{L,R}$. Within our approximations the free energies of the vector and axial-vector currents $\vec{J}_L = \pm\vec{J}_R$ are degenerate. We will consider the pure vector current $\vec{J}_L = \vec{J}_R = \vec{A}^T$ with

$$\vec{A}^T = \tfrac{1}{2}\vec{\jmath}\left(\lambda_3 + \tfrac{1}{\sqrt{3}}\lambda_8 - \tfrac{1}{\sqrt{6}}\lambda_0\right), \tag{2}$$

where λ_A $(A = 1,\ldots,8)$ are the Gell-Mann matrices and $\lambda_0 = \sqrt{\frac{2}{3}}\mathbf{1}$. This ansatz has the feature that it does not shift the energy of electrically charged fermion modes. An ansatz of this type is favored if electric neutrality is enforced. The energetically preferred solution is indeed close to the ansatz equ. (2) [5].

The free energy is given by

$$\Omega = \frac{1}{G}(\Delta_1^2 + \Delta_2^2 + \Delta_3^2) + \frac{3\mu^2\jmath^2}{8\pi^2} - \frac{\mu^2}{4\pi^2}\int dp \int_{-1}^{1} dt \sum_{i=1}^{9}(|\epsilon_i| - |p|), \tag{3}$$

where $p = \vec{v}\cdot\vec{p} - \mu$, $t = \cos\theta$, and ϵ_i are the quasiparticle energies which are obtained from the effective Lagrangian equ. (1). The constant G fixes the magnitude of the gap in the chiral limit, which we denote with $\Delta(0)$.

The integral in equ. (3) is quite complicated. We assume that the external fields and currents are small and expand in a set of parameters $b \in \{\tilde{A}_3/\Delta_1, \tilde{A}_8/\Delta_1, \jmath/\Delta_1, (\Delta_2 - \Delta_1)/\Delta_1, (\Delta_3 - \Delta_1)/\Delta_1, (\mu_s - \Delta_1)/\Delta_1\}$, until numerical convergence is achieved.

We observe that $\Omega/(\mu^2\Delta(0)^2)$ is a function of $b/\Delta(0)$ and $\Delta_1/\Delta(0)$. This means that in order to study the effective potential as a function of dimensionless variables we do not have to specify the value of μ and G. We solve the gap equations and neutrality conditions numerically. Fig. 1 shows $\Omega(\jmath)$ for various values of μ_s. We find that a nontrivial minimum appears for $\mu_s > \mu_{s,crit} = 0.9919\Delta(0)$.

3. MEISSNER MASSES

In this section we study the stability of the current-current correlation functions. The screening (Meissner) masses are given by

$$\begin{aligned}(m_M^2)_{ab}^{ij} &= \frac{g^2\mu^2}{2\pi^2}\delta_{ab}\delta^{ij} + \frac{g^2\mu^2}{16\pi^2}\lim_{\vec{k}\to 0}\lim_{k_0\to 0}\int dp\int_{-1}^{1} dt\int\frac{dp_0}{2\pi} \\ &\quad\times \mathrm{Tr}\Big[G^+(p)V_a^i G^+(p+k)V_b^j + G^-(p)\tilde{V}_a^i G^-(p+k)\tilde{V}_b^j \\ &\quad + \Xi^+(p)V_a^i\Xi^-(p+k)\tilde{V}_b^j + \Xi^-(p)\tilde{V}_a^i\Xi^+(p+k)V_b^j\Big],\end{aligned} \tag{4}$$

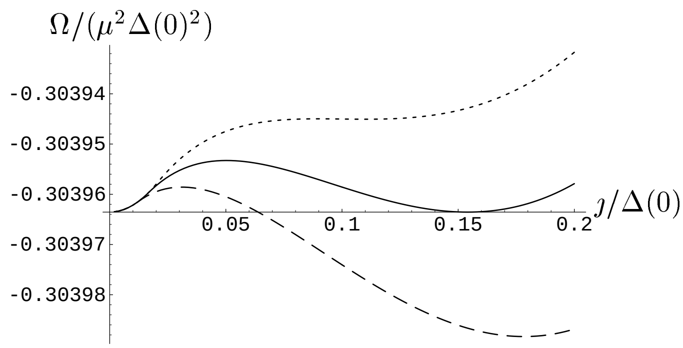

Figure 1. $\Omega/(\mu^2\Delta(0)^2)$ as a function of $\jmath/\Delta(0)$ for $\mu_s = 0.99\Delta(0)$ (dotted), $\mu_s = \mu_{s,crit} = 0.9919\Delta(0)$ (continuous) and $\mu_s = 0.994\Delta(0)$ (dashed).

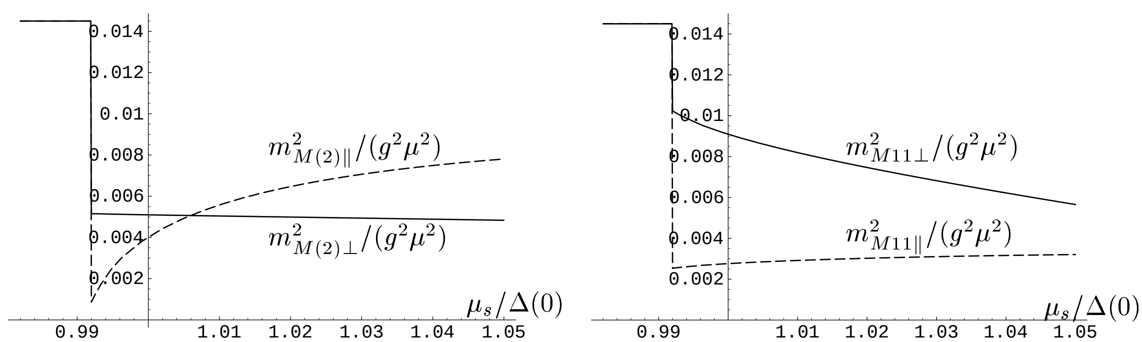

Figure 2. Meissner masses squared $m^2_{M(2)\perp}/(g^2\mu^2)$, $m^2_{M(2)\|}/(g^2\mu^2)$, $m^2_{M11\perp}/(g^2\mu^2)$ and $m^2_{M11\|}/(g^2\mu^2)$ as functions of $\mu_s/\Delta(0)$.

where $G^{\pm}$ and $\Xi^{\pm}$ denote ordinary and anomolous quark propagators [5], and V_a^i and $\tilde{V}_a^i$ are quark-gluon vertices [5].

For $\mu_s < \mu_{s,crit}$ one finds that the Meissner masses are real [4, 5]. In the presence of a finite current $\vec{\jmath}$ we may decompose the Meissner masses into a longitudinal and a transverse component,

$$(m_M^2)^{ij} = m^2_{M\perp}(\delta^{ij} - \hat{\jmath}^i\hat{\jmath}^j) + m^2_{M\|}\hat{\jmath}^i\hat{\jmath}^j. \tag{5}$$

A chromomagnetic instability could occur for the Meissner masses with color indices 1,2,3 or 8 [4]. Fig. 2 shows the “dangerous” components of the Meissner masses as functions of μ_s, where $m^2_{M(2)}$ denotes one of the eigenvalues in the 3-8-sector [5]. We observe that all Meissner masses are real.

4. CONCLUSIONS

We have shown that near the point at which gapless fermion modes appear in the spectrum the CFL phase becomes unstable with respect to the formation of a Goldstone boson current. We have computed the Meissner masses in the Goldstone current phase and found that all masses are real. In this work we have not yet included the effect of a homogeneous kaon condensate, which is an important subject for future studies.

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